

Some impacts of Lorentz violation on cosmology

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ABSTRACT: The impact of Lorentz violation on the dynamics of a scalar field is investigated. In particular, we study the dynamics of a scalar field in the scalar-vector-tensor theory where the vector field is constrained to be unity and time like. By taking a generic form of the scalar field action, a generalized dynamical equation for the scalar-vector-tensor theory of gravity is obtained to describe the cosmological solutions. We present a class of exact solutions for an ordinary scalar field or phantom field corresponding to a power law coupling vector and the Hubble parameter. As the results, we find a constant equation of state in de Sitter space-time and power law expansion with the quadratic of coupling vector, while a dynamic equation of state is obtained for $n > 2$. Then, we consider the inflationary scenario based on the Lorentz violating scalar-vector-tensor theory of gravity with general power-law coupling vector and two typical potentials: inverse power-law and power-law potentials. In fact, both the coupling vector and the potential models affect the dynamics of the inflationary solutions. Finally, we use the dynamical system formalism to study the attractor behavior of a cosmological model containing a scalar field endowed with a quadratic coupling vector and a chaotic potential.

KEYWORDS: Cosmology of Theories beyond the SM, Classical Theories of Gravity.

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1. Introduction

Scalar field theory has become the generic playground for building cosmological models related to particle physics, in particular for obtaining inflationary cosmologies which is one of the most reliable concepts to describe the early stage of the universe. The key property of the laws of physics that makes inflation possible is the existence of states of matter that have a high energy density which cannot be rapidly lowered. The Inflationary scenario [1] relies on the potential energy of a scalar (inflaton) field to drive a period of early universe acceleration. It has been thought that the early universe could be well characterized by a series of phase transitions, in which topological defects could be formed [2]. In the context of the string theory, the natural values of the gauge and gravitational couplings in our 4d universe are explained by the dynamics of 'moduli' scalar fields [3, 4]. Moreover, originating from the work of Sen [5] (see also [6]), the possibility of the tachyon field being a candidate for the inflaton has been extensively studied. The tachyon action is of the Dirac Born Infeld form [7] which leads to an equation of state interpolating between -1 at early times and 0 at late-times. This suggests the possibility that the tachyon can play the role of the inflaton at early times and the dark matter at late-times. Serious difficulties, however,

plagues the tachyonic inflation [8]. These include large density perturbations, problem with reheating and formation of caustics.

Recent observational evidences especially from the Type Ia Supernovae [9, 10] and WMAP satellite missions [11], indicate that we live in a favored spatially flat universe consisting approximately of 30% dark matter and 70% dark energy. In the framework of the General Relativity this means that about two thirds of the total energy density of the Universe consists of dark energy, the still unknown component with a relativistic negative pressure $p < -\rho/3$. The simplest candidate for dark energy is the cosmological Λ -term. During the cosmological evolution the Λ -term component has the constant (Lorentz invariant) energy density ρ and pressure $p = -\rho$. However, it has got the famous and serious fine-tuning problem, while the also elusive dark matter candidate might be a lightest and neutral supersymmetry particle with only gravity interaction. For this reason the different forms of dynamically changing dark energy with an effective equation of state $w < -1/3$ were proposed, instead of the constant vacuum energy density. As a particular example of dark energy, the scalar field with a slow rolling potential (quintessence) [12] is often considered. The possible generalization of quintessence is a k-essence [13], the scalar field with a non-canonical Lagrangian. Any such behavior would have far-reaching implications for particle physics. However, recent theory of gravity with the Lorentz violation are proposed. The cosmological feature of the Lorentz violation are nowadays being investigated even more than particle physics consequences and to the major problems recent astrophysical data have revealed: the presence of high energy cosmic rays about the Greisen-Zatsepin-Kuzmin cutoff [14–16], the dark matter and the dark energy problem, the inflation, the baryogenesis, cosmic rays, and so on [17–19]. This may suggests the Lorentz violation [20, 21]. Theoretically, any quantum theory of gravity requires drastic modification of the picture of space-time at the Planck scale [22, 23].

More recently, authors in ref. [24] explored the Lorentz violating scenario in the context of the scalar-vector-tensor theory. They showed that the Lorentz violating vector affects the dynamics of the inflationary model. One of the interesting features of this scenario, is that the exact Lorentz violating inflationary solutions are related to the absence of the inflaton potential. In this case, the inflation is completely associated with the Lorentz violation. Depending on the value of the coupling parameter, the three kind of exact solutions are found: the power law inflation, de Sitter inflation, and the super-inflation. Moreover, the spontaneous breaking of Lorentz symmetry due to a vector field [23, 25–27] has a wide parameter region where all of current experiments and observations can be explained [28–32].

The purpose of this paper is to study the dynamics of a scalar field in the framework of Lorentz violating scalar-vector-tensor model, taking into account the effect of the dynamically coupling vector. In this framework, we explore a class of exact solutions such as evolution of a scalar field and equation of state parameters. We discuss an inflationary scenario with a power-law coupling vector model with two typical potentials: an inverse power-law potential and a power-law potential. Then, we show that it is possible to find attractor solutions in the Lorentz violating scalar-vector-tensor model in which both the coupling function and the potential function are specified.

The organization of this paper is the following: in section 2, we set down the general formalism for the scalar-vector-tensor theory where the Lorentz symmetry is spontaneously broken due to the unit-norm vector field. In section 3, we use our formalism to find an exact solution of the equation of state. In section 4, we study Lorentz violating (inverse) power law inflation. In section 5, the critical points of the global system and their stability are presented. The final section is devoted to the conclusions.

2. General formalism

In the present section, we develop the general reconstruction scheme for the scalar-vector-tensor theory. We will consider the properties of general four-dimensional universe, i.e. the universe where the four-dimensional space-time is allowed to contain any non-gravitational degree of freedom in the framework of Lorentz violating scalar-tensor-vector theory of gravity. Let us assume that there exists the Lorentz symmetry but it is spontaneously broken by getting the expectation values of a vector field u^μ as $\langle 0|u^\mu u_\mu|0 \rangle = -1$. The action can be written as the sum of three distinct parts:

$$S = S_g + S_u + S_\phi, \tag{2.1}$$

where the actions for the tensor field S_g , the vector field S_u , and the scalar field S_ϕ are,

$$S_g = \int d^4x \sqrt{-g} \frac{1}{16\pi G} R, \tag{2.2}$$

$$S_u = \int d^4x \sqrt{-g} \left[-\beta_1 \nabla^\mu u^\nu \nabla_\mu u_\nu - \beta_2 \nabla^\mu u^\nu \nabla_\nu u_\mu - \beta_3 (\nabla_\mu u^\mu)^2 - \beta_4 u^\mu u^\nu \nabla_\mu u^\alpha \nabla_\nu u_\alpha + \lambda (u^\mu u_\mu + 1) \right], \tag{2.3}$$

$$S_\phi = \int d^4x \sqrt{-g} \mathcal{L}_\phi. \tag{2.4}$$

In the above $\beta_i(\phi)$ ($i = 1, 2, 3, 4$) are arbitrary parameters and \mathcal{L}_ϕ is the Lagrangian density for scalar field, expressed as a function of the metric $g_{\mu\nu}$ and the scalar field ϕ . Then, the action (2.1) describes the scalar-tensor-vector theory of gravity. Here we have taken into account the expectation value by just adopting it as a constraint

$$u^\mu u_\mu = -1. \tag{2.5}$$

We take u^μ as the dimensionless vector, and accordingly β_i has the dimension of mass squared. In other words, $\sqrt{\beta_i}$ gives the mass scale of symmetry breakdown. In principle, the preferred frame determined by the vector u^μ differs from the CMB rest frame. Note that the alignment of these frames had been achieved during the cosmic expansion as is explained in the appendix of ref. [24]. In this setup, the preferred frame is selected through the constrained vector field u^μ and this leads to the violate Lorentz symmetry.

For the background solutions, we use the homogeneity and isotropy of the universe spacetime

$$ds^2 = -\mathcal{N}^2(t) dt^2 + e^{2\alpha(t)} \delta_{ij} dx^i dx^j, \tag{2.6}$$

where \mathcal{N} is a lapse function. The scale of the universe is determined by α . We take the constraint

$$u^\mu = \left(\frac{1}{\mathcal{N}}, 0, 0, 0 \right), \quad (2.7)$$

where $\mathcal{N} = 1$ is taken into account after the variation. Varying the action (2.1) with respect to $g^{\mu\nu}$, we have field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu}, \quad (2.8)$$

where $T_{\mu\nu} = T_{\mu\nu}^{(u)} + T_{\mu\nu}^{(\phi)}$ is the total energy-momentum tensor, $T_{\mu\nu}^{(u)}$ and $T_{\mu\nu}^{(\phi)}$ are the energy-momentum tensors of vector and scalar fields, respectively, defined by the usual formulae

$$T_{\mu\nu}^{(k)} = -2 \frac{\partial \mathcal{L}^{(k)}}{\partial g^{\mu\nu}} + g_{\mu\nu} \mathcal{L}^{(k)}, \quad k = u, \phi. \quad (2.9)$$

The time and space components of the total energy-momentum tensor are given by

$$T_0^0 = -\rho_u - \rho_\phi, \quad T_i^i = p_u + p_\phi, \quad (2.10)$$

where the energy density and pressure of the vector field are given by

$$\rho_u = -3\beta H^2, \quad (2.11)$$

$$p_u = \left(3 + 2\frac{H'}{H} + 2\frac{\beta'}{\beta} \right) \beta H^2, \quad (2.12)$$

$$\beta \equiv \beta_1 + 3\beta_2 + \beta_3. \quad (2.13)$$

Note that β_4 does not contribute to the background dynamics. A prime denotes the derivative of any quantities X with respect to α . X' is then related to its derivative with respect to t by $X' = (dX/dt)H^{-1} = \dot{X}H^{-1}$ where $H = d\alpha/dt = \dot{\alpha}$ is the Hubble parameter. From eqs. (2.11) and (2.12), one obtains the energy equation for the vector field u

$$\rho'_u + 3(\rho_u + p_u) = +3H^2\beta', \quad (2.14)$$

and for the scalar field

$$\rho'_\phi + 3(\rho_\phi + p_\phi) = -3H^2\beta'. \quad (2.15)$$

The total energy equation in the presence of both the vector and the scalar fields is, accordingly,

$$\rho' + 3(\rho + p) = 0, \quad (\rho = \rho_u + \rho_\phi). \quad (2.16)$$

This energy conservation equation can also be obtained by equating the covariant divergence of the total energy-momentum tensor to zero, since the covariant divergence of the Einstein tensor is zero by its geometric construction. It follows from contraction of the geometric Bianchi identity.

Substituting eq. (2.10) into the Einstein equations (2.8), we obtain two independent equations, called the Friedmann equations, as follows:

$$-3H^2 = 8\pi G (3\beta H^2 - \rho_\phi) , \quad (2.17)$$

$$-2HH' - 3H^2 = 8\pi G \left[\left(3 + 2\frac{H'}{H} + 2\frac{\beta'}{\beta} \right) \beta H^2 + p_\phi \right] . \quad (2.18)$$

These Friedmann equations can be rewritten as

$$\left(1 + \frac{1}{8\pi G\beta} \right) H^2 = \frac{1}{3\beta} \rho_\phi , \quad (2.19)$$

$$\left(1 + \frac{1}{8\pi G\beta} \right) (HH' + H^2) = -\frac{1}{6} \left(\frac{\rho_\phi}{\beta} + \frac{3p_\phi}{\beta} \right) - H^2 \frac{\beta'}{\beta} . \quad (2.20)$$

The second term on r.h.s. of eq. (2.20) is a consequence of the coupling vector field as a function of scalar field. If $\beta_i = 0$, thus without the vector field, the above equations reduce to the conventional ones. And in the case $\beta = const.$, the above equations are lead to the Friedmann equations given in ref. [18].

Using eqs. (2.19) and (2.15), we obtain a set of equations as follows:

$$\frac{H'}{H} + \frac{\bar{\beta}'}{\bar{\beta}} + \frac{3}{2}(1 + \omega_\phi) = 0 , \quad (2.21)$$

$$\frac{H'}{H} - \frac{\rho'_\phi}{\rho_\phi} - \frac{3}{2}(1 + \omega_\phi) = 0 , \quad (2.22)$$

$$\frac{\rho'_\phi}{\rho_\phi} + \frac{\bar{\beta}'}{\bar{\beta}} + 3(1 + \omega_\phi) = 0 , \quad (2.23)$$

where

$$\bar{\beta} = \beta + \frac{1}{8\pi G} , \quad (2.24)$$

and $\omega_\phi = p_\phi/\rho_\phi$ is the equation of state of the scalar field. It is easy to check that the equations (2.21)–(2.23) satisfy the following constraint

$$2\frac{H'}{H} + \frac{\bar{\beta}'}{\bar{\beta}} - \frac{\rho'_\phi}{\rho_\phi} = 0 . \quad (2.25)$$

In order to solve the eqs. (2.21)–(2.23) and (2.25), we have to specify the model and the matter content of the universe. The general solution of these equations can be written as

$$H\bar{\beta} \propto \exp \left[- \int \frac{3}{2}(1 + \omega_\phi(\alpha))d\alpha \right] , \quad (2.26)$$

$$\frac{H}{\rho_\phi} \propto \exp \left[\int \frac{3}{2}(1 + \omega_\phi(\alpha))d\alpha \right] , \quad (2.27)$$

$$\rho_\phi\bar{\beta} \propto \exp \left[- \int 3(1 + \omega_\phi(\alpha))d\alpha \right] . \quad (2.28)$$

If the functions ω_ϕ and $\bar{\beta}$ are given, then we can find the evolution of the Hubble parameter under the Lorentz violation. For example, the cosmological constant corresponds to a fluid

with a constant equation of state $\omega_\phi = -1$. Thus the above equations reduce to: $H\bar{\beta} \propto 1$, $H \propto \rho_\phi$ and $\rho_\phi\bar{\beta} \propto 1$ where H , ρ_ϕ and β are functions of α . If ω_ϕ is a constant parameter of a simple one component fluid, and for a given $\alpha(t)$, eqs. (2.21)–(2.23) can be used to determine $\beta(\alpha)$ and $\rho_\phi(\alpha)$. We, then, are able to determine the potential of the Lorentz violation model.

3. Dynamical equations for scalar fields

For a given scalar field Lagrangian with the FRW background, we can obtain the equations of motion for a scalar field by using eq. (2.15) and eqs. (2.21)–(2.23). Let us consider the Lagrangian density of a scalar field ϕ with a potential $V(\phi)$ in eq. (2.1):

$$\mathcal{L}_\phi = -\frac{\eta}{2}(\nabla\phi)^2 - V(\phi), \tag{3.1}$$

where $(\nabla\phi)^2 = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. Ordinary scalar fields correspond to $\eta = 1$ while $\eta = -1$ is for phantoms. For the homogeneous field the density ρ_ϕ and pressure p_ϕ of the scalar field, may be found as follows

$$\rho_\phi = \frac{\eta}{2}H^2\phi'^2 + V(\phi), \tag{3.2}$$

$$p_\phi = \frac{\eta}{2}H^2\phi'^2 - V(\phi). \tag{3.3}$$

The corresponding equation of state parameter is, accordingly

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = -\frac{1 - \eta H^2\phi'^2/2V}{1 + \eta H^2\phi'^2/2V}. \tag{3.4}$$

Substituting eq. (3.2) into eq. (2.19), the Friedmann equation leads to

$$H^2 = \frac{1}{3\bar{\beta}} \left[\frac{\eta}{2}H^2\phi'^2 + V(\phi) \right]. \tag{3.5}$$

Now, differentiating eq. (3.2) with respect to α and using eq. (2.15), and also differentiating eq. (3.5) with respect to α and using eq. (3.6) give, respectively,

$$\phi'' = -\left(\frac{H'}{H} + 3\right)\phi' - \eta\frac{V_{,\phi}}{H^2} - 3\eta\bar{\beta}_{,\phi}, \tag{3.6}$$

$$\phi' = -2\eta\bar{\beta}\left(\frac{H_{,\phi}}{H} + \frac{\bar{\beta}_{,\phi}}{\bar{\beta}}\right). \tag{3.7}$$

Substituting eq. (3.7) into the Friedmann equation the potential of the scalar field can be written as

$$V = 3\bar{\beta}H^2 \left[1 - \frac{2}{3}\eta\bar{\beta}\left(\frac{\bar{\beta}_{,\phi}}{\bar{\beta}} + \frac{H_{,\phi}}{H}\right)^2 \right]. \tag{3.8}$$

Note that in the above equations the Hubble parameter H has been expressed as a function of ϕ , $H = H(\phi(t))$. From eq. (2.21), the equation of state can be written as

$$\begin{aligned}\omega_\phi &= -1 + \frac{4}{3}\eta\bar{\beta}\left(\frac{H_{,\phi}}{H} + \frac{\bar{\beta}_{,\phi}}{\bar{\beta}}\right)^2 \\ &= -1 + \frac{1}{3}\eta\frac{\phi'^2}{\bar{\beta}}.\end{aligned}\tag{3.9}$$

Equations (3.7) and (3.9) are two equations that we need to solve for the scalar field ϕ and the equation of state ω_ϕ . This is achieved only if the Hubble parameter $H(\phi)$ and the coupling vector $\bar{\beta}(\phi)$ are known. For different choice of the Hubble parameter $H(\phi)$ and the coupling vector $\bar{\beta}(\phi)$, it is possible to extract a class of exact solutions of eqs. (3.7) and (3.9). We shall solve eqs. (3.7) and (3.9) to obtain the following physical quantities (V and K are the potential and kinetic energies, respectively):

$$\begin{aligned}V &= \frac{3}{2}(1 - \omega_\phi)\bar{\beta}H^2, & K &= \frac{3}{2}(1 + \omega_\phi)\bar{\beta}H^2, \\ \rho_\phi &= 3\bar{\beta}H^2, & p_\phi &= 3\omega_\phi\bar{\beta}H^2.\end{aligned}\tag{3.10}$$

In the following two subsections we will explore a class of exact solutions.

3.1 Exact solutions and the behavior of scalar fields

We shall have to solve equations (3.7) and (3.9) for H , ω_ϕ , $\bar{\beta}$, and V , which is not possible unless two are known. In the present subsection, we consider an example to find an exact solution of the equation of state of the scalar field in the quadratic coupling vector. The equation of state for the scalar field has been intensively studied in [33] for the so called tracking cosmological solutions introduced in [34], and some classes of potentials allowing for the field equation of state were described.

Let us consider a simple model

$$H = H_0, \quad \bar{\beta}(\phi) = m\phi^2,\tag{3.11}$$

where H_0 and m are positive constant parameters. The equation (3.7) can now be integrated to yield the evolution of the scalar field

$$\phi(t) = \phi_0 \exp[-4\eta m H_0(t - t_0)],\tag{3.12}$$

where $\phi(t = t_0) \equiv \phi_0$ is a constant. Then, it is easy to find the equation of state of the scalar field by using eq. (3.9). We obtain

$$\omega_\phi = -1 + \frac{16}{3}m, \quad \text{for ordinary scalar fields},\tag{3.13}$$

$$\omega_\phi = -1 - \frac{16}{3}m, \quad \text{for phantom fields}.\tag{3.14}$$

Then, the potential and the kinetic energies, the energy density and the pressure of the scalar field evolve according to

$$V(t) = mH_0^2\phi_0^2(3 - 8m) \exp[-8\eta mH_0(t - t_0)] , \quad (3.15)$$

$$K(t) = 8\eta (mH_0\phi_0)^2 \exp[-8\eta mH_0(t - t_0)] , \quad (3.16)$$

$$\rho(t) = 3mH_0^2\phi_0^2 \exp[-8\eta mH_0(t - t_0)] , \quad (3.17)$$

$$p(t) = \eta mH_0^2\phi_0^2 (16m - 3\eta) \exp[-8\eta mH_0(t - t_0)] . \quad (3.18)$$

The above solutions are completely associated with the Lorentz violation. The model (3.11) depicts that the cosmic evolution starts from a constant value of the scale factor and grows exponentially, $a(t) = a_0 e^{H_0(t-t_0)}$. The coupling vector decreases exponentially for the ordinary scalar field and increases exponentially for the phantom field from a constant value of $m\phi_0^2$. Hence the potential energy, the kinetic energy, the energy density and the pressure decrease exponentially for the ordinary scalar field. For the phantom field, on the other hand, the potential energy and energy density as well as the absolute values of the kinetic energy and the pressure increase exponentially. Note that the kinetic energy and the pressure begin with the negative values. The eqs. (3.13)–(3.14) show that the equation of state ω_ϕ is non-dynamical because it only depends on the value of the coupling vector parameter m , both for the ordinary scalar and the phantom fields. Since an accelerated expansion occurs for $\omega_\phi < -1/3$ then we have $m < 1/8$ for the ordinary scalar field. However, the present data of the Universe seems to tell that ω_ϕ might be less than -1 . Thus, the value m may be chosen in order to fit the present observable constraint on the equation of state parameter.

In other case, for instance, $H(\phi) = H_0\phi^\xi$ and $\bar{\beta}(\phi) = m\phi^2$, we also find the constant equation of state,

$$\omega_\phi = -1 + \frac{4}{3}\eta m(\xi + 2)^2 . \quad (3.19)$$

The condition for the accelerating Universe \ddot{a} or $H'/H > -1$ yields

$$\eta m < \frac{1}{2\xi(\xi + 2)} . \quad (3.20)$$

This model gives a power law expansion

$$\frac{a(t)}{a_o} = \left[1 + \frac{H_0\phi_0^\xi}{p}(t - t_0) \right]^p , \quad p > 1 , \quad (3.21)$$

where

$$p = \frac{1}{2\eta m\xi(\xi + 2)} . \quad (3.22)$$

The scalar field evolve as

$$\phi(t) = \phi_0 \left(1 + \frac{H_0\phi_0^\xi}{p}(t - t_0) \right)^{-1/\xi} . \quad (3.23)$$

Hence, the complete set of solutions is found by substituting eqs. (3.19) and (3.23) into eqs. (3.10).

In the following subsection, we will see that the equation of state may be dynamics. For this purpose we generalize the coupling vector to $\bar{\beta}(\phi) = m\phi^n$, $n > 2$.

3.2 Variable equation of states

Let us consider a model where the coupling vector is a power law of the scalar field,

$$H = H_0, \quad \bar{\beta}(\phi) = m\phi^n, \quad n > 2, \quad (3.24)$$

where H_0 , m and n are constant positive parameters. Following the same above procedure, the scalar ϕ can be evaluated as,

$$\phi(t) = \frac{\phi_0}{[1 + 2\eta mn H_0 (n-2) \phi_0^{n-2} (t-t_0)]^{\frac{1}{n-2}}}, \quad (3.25)$$

the coupling vector is given by

$$\bar{\beta}(t) = \frac{m\phi_0^n}{[1 + 2\eta mn H_0 (n-2) \phi_0^{n-2} (t-t_0)]^{\frac{n}{n-2}}}, \quad (3.26)$$

and the dynamical equation of state (3.9) is

$$\omega(t) = -1 + \frac{4\eta mn^2 \phi_0^{n-2} / 3}{1 + 2\eta mn H_0 (n-2) \phi_0^{n-2} (t-t_0)}. \quad (3.27)$$

Then, the potential and kinetic energies, the energy density and the pressure of the scalar field are given by

$$V(t) = 3mH_0^2\phi_0^n \left[1 - \frac{2\eta mn^2 \phi_0^{n-2} / 3}{1 + 2\eta mn H_0 (n-2) \phi_0^{n-2} (t-t_0)} \right] \times \frac{1}{[1 + 2\eta mn H_0 (n-2) \phi_0^{n-2} (t-t_0)]^{\frac{n}{n-2}}}, \quad (3.28)$$

$$K(t) = \frac{2\eta (mnH_0\phi_0^{n-1})^2}{[1 + 2\eta mn H_0 (n-2) \phi_0^{n-2} (t-t_0)]^{\frac{2(n-1)}{n-2}}}, \quad (3.29)$$

$$\rho(t) = \frac{3mH_0^2\phi_0^n}{[1 + 2\eta mn H_0 (n-2) \phi_0^{n-2} (t-t_0)]^{\frac{n}{n-2}}}, \quad (3.30)$$

$$p(t) = 3mH_0^2\phi_0^n \left[-1 + \frac{4\eta mn^2 \phi_0^{n-2} / 3}{1 + 2\eta mn H_0 (n-2) \phi_0^{n-2} (t-t_0)} \right] \times \frac{1}{[1 + 2\eta mn H_0 (n-2) \phi_0^{n-2} (t-t_0)]^{\frac{n}{n-2}}}. \quad (3.31)$$

Thus, the model (3.24) describes that the cosmic evolution grows exponentially from a constant value of the scale factor, $a(t) = a_0 e^{H_0(t-t_0)}$, while the coupling vector $\bar{\beta}$ started from a constant value of the scalar field, $m\phi_0^n$. The equation of state ω_ϕ is dynamical both

for the ordinary scalar and phantom fields. Then the potential energy, kinetic energy, the energy density and the pressure decrease for the ordinary scalar field. For the phantom field, on the other hand, the potential and energy density increase while the kinetic energy and pressure begin with the negative values.

4. Lorentz violating inflation scenario

As it has been studied by authors in ref. [24], the Lorentz violation on the inflationary scenario can be divided into two parts: the Lorentz violations stage $8\pi G\beta \gg 1$ and the standard slow roll stage $8\pi G\beta \ll 1$. The first stage corresponds to $\bar{\beta} = \beta$ in eq. (2.24) and the second stage corresponds to $\bar{\beta} = 1/8\pi G$, then we have the usual dynamical equations.

In this section we will consider the inflationary scenario for the scalar field (inflaton). In particular, we consider a power-law coupling vector, $\beta(\phi) = m\phi^n$, with two types of the potential: $V(\phi) = \mu^{4+\nu}\phi^{-\nu}$ and $V(\phi) = \frac{1}{2}M^2\phi^2$. Here μ, ν and M are parameters. Thus, the dynamics of each particular inflationary model are determined by the Friedmann equation and the scalar field equation of motion once the functional form of the inflaton potential and the coupling parameter have been specified. Let us collect the dynamics-related equations for the inflaton the Friedmann equation (3.5) in inflationary models

$$H^2 = \frac{1}{3\bar{\beta}} \left[\frac{1}{2}H^2\phi'^2 + V(\phi) \right], \tag{4.1}$$

the constraint equation (obtained from eqs. (3.4) and (2.21))

$$\frac{H'}{H} + \frac{1}{2} \frac{\phi'^2}{\bar{\beta}} + \frac{\bar{\beta}'}{\bar{\beta}} = 0, \tag{4.2}$$

and the equation of motion eq. (3.6)

$$\phi'' + \frac{H'}{H}\phi' + 3\phi' + \frac{V_{,\phi}}{H^2} + 3\bar{\beta}_{,\phi} = 0. \tag{4.3}$$

$\bar{\beta}$ is given by eq. (2.24). The above equation of motion change its property at the critical value ϕ_c defined by

$$8\pi G\beta(\phi_c) = 1. \tag{4.4}$$

For example, a coupling parameter of the form $\beta = m\phi^2$ gives the critical value

$$\phi_c = \frac{M_{pl}}{\sqrt{8m\pi}}, M_{pl} = G^{-1}. \tag{4.5}$$

Let ϕ_i be the corresponding initial value of the scalar field. Putting $\phi_i \sim 3M_{pl}$, the Lorentz violation implies the criterion $m > 1/(72\pi) \sim 1/226$.

The set of eqs. (4.1)–(4.3) constitutes the equations we have to solve for the problem specified by the coupling parameter $\beta(\phi)$ and the potential $V(\phi)$. In following subsection, we consider with a model with the coupling parameter $\beta(\phi)$ is given by

$$\beta(\phi) = m\phi^n, \tag{4.6}$$

where n and m are parameters. For the model (4.6), we obtain the critical value of the scalar field and the criterion for Lorentz violation

$$\phi_c = \left(\frac{M_{pl}^2}{8m\pi} \right)^{1/n} \quad \text{and} \quad m > \frac{M_{pl}^2}{8\pi(3M_{pl})^n} . \quad (4.7)$$

Now, we consider two typical potentials appear in many cosmological implications: an inverse power-law potential and a power-law potential. We discuss those solutions and analyze the two regimes separately.

4.1 Inverse power law potential: $V(\phi) = \mu^{4+\nu}\phi^{-\nu}$

In this subsection, we consider the class of power law potential

$$V(\phi) = \mu^{4+\nu}\phi^{-\nu} , \quad (4.8)$$

where μ and ν are constants. Inverse power law models are interesting for a number of reasons. In conventional cosmology, they drive intermediate inflation [35] and typically produce significant tensor perturbations for almost scale-invariant scalar fluctuations. They arise in supersymmetric condensate models of QCD [36] and can in principle act as a source of quintessence [12, 37].

4.1.1 Lorentz violating stage

For a sufficiently larger value of ϕ , both the coupling function β and the potential function V are important in the present model. During this period, the effect of Lorentz violation on the inflaton dynamics must be large. In the Lorentz violating regime, $8\pi G\beta \gg 1$ ($\bar{\beta} = \beta$), we have the equation (4.1)–(4.3).

An inflationary epoch, in which the scale factors a are accelerating, requires the scalar field ϕ to evolve slowly compared to the expansion of the universe. Thus, the following conditions of slow-rolling are required:

$$H^2\phi'^2 \ll V, \quad \phi'' \ll \phi', \quad \phi'^2 \ll \beta, \quad \text{and} \quad \beta' \ll \beta . \quad (4.9)$$

The formalism which gives these slow roll conditions are discussed in ref. [24]. This is sufficient to guarantee inflation. Under the slow-roll conditions eq. (4.9), the eqs. (4.1)–(4.3) can be simplified. We obtain the slow roll equations

$$H^2 \simeq \frac{V}{3\beta}, \quad \text{and} \quad \phi' \simeq -\beta \left(\frac{\beta_{,\phi}}{\beta} + \frac{V_{,\phi}}{V} \right) . \quad (4.10)$$

Inserting eq. (4.6) and the potential of the form $V(\phi) = \mu^{4+\nu}\phi^{-\nu}$ into eq. (4.10), we have

$$H^2 = \frac{\mu^{4+\nu}}{3m} \phi^{-(\nu+n)} , \quad (4.11)$$

$$\phi' = -m(n - \nu)\phi^{(n-1)} . \quad (4.12)$$

One can then solve for ϕ from eq. (4.12),

$$\phi(\alpha) = [\phi_i^{2-n} + m(n-2)(n-\nu)(\alpha - \alpha_i)]^{-\frac{1}{n-2}} , \quad \text{for} \quad n \neq 2, n \neq \nu , \quad (4.13)$$

where $\phi(\alpha = \alpha_i) \equiv \phi_i$ is a constant. The Friedmann equation gives

$$H^2(\alpha) = \frac{\mu^{4+\nu}}{3m} [\phi_i^{2-n} + m(n-2)(n-\nu)(\alpha - \alpha_i)]^{\frac{n+\nu}{n-2}}. \quad (4.14)$$

The solution (4.13) and the slow roll conditions (4.9) during the Lorentz violating stage give $n > 2$ because

$$\phi'^2 \left(\sim \alpha^{-2(1-n)/(2-n)} \right) \ll \beta \left(\sim \alpha^{n/(2-n)} \right), \quad (4.15)$$

$$\beta' \left(\sim \alpha^{-2(1-n)/(2-n)} \right) \ll \beta \left(\sim \alpha^{n/(2-n)} \right). \quad (4.16)$$

From eq. (4.14), the universe expands during the Lorentz violating stage as

$$\frac{a(t)}{a_i} = \exp \left\{ -\frac{B}{C} + \frac{1}{C} \left(\frac{1}{B^{D-1}} - AC(D-1)(t-t_i) \right)^{-\frac{1}{D-1}} \right\}, \quad C \neq 0, \quad (4.17)$$

where the constants A, B, C and D are

$$A = \sqrt{\frac{\mu^{4+\nu}}{3m}}, \quad B = \phi_i^{2-n}, \quad C = m(n-2)(n-\nu), \quad D = \frac{n+\nu}{2(n-2)}. \quad (4.18)$$

Combining eqs. (4.17), (4.13) and (4.14), we obtain the physical quantities

$$\phi(t) = \left(\frac{1}{B^{D-1}} - AC(D-1)(t-t_i) \right)^{\frac{1}{(D-1)(n-2)}}, \quad (4.19)$$

$$H(t) = A \left(\frac{1}{B^{D-1}} - AC(D-1)(t-t_i) \right)^{-\frac{D}{(D-1)}}, \quad (4.20)$$

$$\beta(t) = n \left(\frac{1}{B^{D-1}} - AC(D-1)(t-t_i) \right)^{\frac{n}{(D-1)(n-2)}}. \quad (4.21)$$

The scalar field energy density, on the other hand, evolves according to

$$\rho(t) \simeq V = 3mA^2 \left(\frac{1}{B^{D-1}} - AC(D-1)(t-t_i) \right)^{\frac{n-2D(n-2)}{(D-1)(n-2)}}. \quad (4.22)$$

One can see that the Hubble parameter H decreases during the Lorentz violation stage.

For $n = 2, \nu \neq 2$, eq. (4.6) and the second part of eq. (4.10) gives

$$\phi(\alpha) = \phi_i e^{-n(2-\nu)(\alpha-\alpha_i)}, \quad (4.23)$$

where $\phi(\alpha = \alpha_i) \equiv \phi_i$. For this solution to satisfy slow roll conditions (4.9), we need $m < 1/(2-\nu)^2$. Thus, we have the range $1/226 < m < 1/(2-\nu)^2$ of the parameter for which the Lorentz violating inflation is relevant. The Hubble parameter as a function of the scale factor, α , is given by

$$H^2(\alpha) = H_i^2 e^{-m(\nu^2-4)(\alpha-\alpha_i)}, \quad (4.24)$$

where

$$H_i^2 = \frac{\mu^{4+\nu}}{3m\phi_i^{(2+\nu)}}, \quad (4.25)$$

while the scale factor $a(t) = e^\alpha$ is of the form:

$$\frac{a(t)}{a_i} = \left[\frac{\phi(t)}{\phi_i} \right]^{\frac{1}{m(\nu-2)}}. \quad (4.26)$$

Now we obtain the evolution of some physical quantities as follows

$$\frac{a(t)}{a_i} = [m(\nu^2 - 4)H_i^2(t - t_i)]^{\frac{1}{m(\nu^2-4)}}, \quad (4.27)$$

$$\frac{\phi(t)}{\phi_i} = [m(\nu^2 - 4)H_i^2(t - t_i)]^{\frac{1}{\nu+2}}, \quad (4.28)$$

$$\frac{H(t)}{H_i^2} = \sqrt{m(4 - \nu^2)(t - t_i)}, \quad (4.29)$$

$$\frac{\beta(t)}{m\phi_i^2} = [m(\nu^2 - 4)H_i^2(t - t_i)]^{\frac{2}{\nu+2}}, \quad (4.30)$$

$$\rho(t) = \frac{\mu^{4+\nu}}{\phi_i} [m(\nu^2 - 4)H_i^2(t - t_i)]^{-\frac{\nu}{\nu+2}}. \quad (4.31)$$

4.1.2 Standard slow roll stage

After the inflaton crosses the critical value ϕ_c , the dynamics is governed entirely by the potential V . In the standard slow roll regime $8\pi G\beta \ll 1$ ($\bar{\beta} = (8\pi G)^{-1}$), we have

$$H^2 = \frac{8\pi G}{3} \left[\frac{1}{2}H^2\phi'^2 + V \right], \quad (4.32)$$

$$\frac{H'}{H} + 4\pi G\phi'^2 = 0, \quad (4.33)$$

$$\phi'' + \frac{H'}{H}\phi' + 3\phi' + \frac{V_{,\phi}}{H^2} = 0. \quad (4.34)$$

The usual slow roll conditions give the slow roll equations

$$H^2 \simeq \frac{8\pi G}{3}V, \quad \phi' \simeq -\frac{1}{8\pi G} \frac{V_{,\phi}}{V}. \quad (4.35)$$

In the simplest case $V(\phi) = \mu^{4+\nu}\phi^{-\nu}$, the evolution of the inflaton and the Hubble parameter can be solved as

$$\phi^2(\alpha) = \phi_c^2 + \frac{\nu}{4\pi G}(\alpha - \alpha_c), \quad (4.36)$$

$$H^2(\alpha) = \frac{8\pi G}{3}\mu^{4+\nu} \left[\phi_c^2 + \frac{\nu}{4\pi G}(\alpha - \alpha_c) \right]^{-\nu/2}, \quad (4.37)$$

and the scale factor is given by

$$\frac{a(t)}{a_c} = \exp \left[\frac{4\pi G}{\nu}(\phi^2(t) - \phi_c^2) \right]. \quad (4.38)$$

The standard inflation stage ends and the reheating commences when the slow roll conditions violate. The evolution equations are given by

$$\frac{a(t)}{a_c} = \exp \left\{ -\frac{B_s}{C_s} + \frac{1}{C_s} [B_s^{D_s+1} + A_s C_s (D_s + 1)(t - t_c)]^{\frac{1}{D_s+1}} \right\}, \quad (4.39)$$

$$\phi(t) = [B_s^{D_s+1} + A_s C_s (D_s + 1)(t - t_c)]^{\frac{1}{2(D_s+1)}}, \quad (4.40)$$

$$H(t) = A_s [B_s^{D_s+1} + A_s C_s (D_s + 1)(t - t_c)]^{\frac{D_s}{(D_s+1)}}, \quad (4.41)$$

and the scalar field energy density evolves as

$$\rho(t) = \frac{3}{8} \left(\frac{A_s^2 C_s}{D_s} \right) [B_s^{D_s+1} + A_s C_s (D_s + 1)(t - t_c)]^{\frac{2D_s}{(D_s+1)}}. \quad (4.42)$$

where A_s , B_s , C_s and D_s are the constants,

$$A_s = \sqrt{\frac{8\pi G}{3}} \mu^{4+\nu}, \quad B_s = \phi_c^2, \quad C_s = \frac{\nu}{4\pi G}, \quad D_s = \frac{\nu}{4}. \quad (4.43)$$

Note that, in the standard slow roll stage, the Hubble parameter H increases.

Another interesting quantity is the number of e-folding during the inflationary phase. The total e-folding number reads

$$\begin{aligned} N &= -\frac{B}{C} + \frac{1}{C} \left(\frac{1}{B^{D-1}} - AC(D-1)(t_c - t_i) \right)^{-\frac{1}{D-1}} \\ &\quad - \frac{B_s}{C_s} + \frac{1}{C_s} [B_s^{D_s+1} + A_s C_s (D_s + 1)(t_e - t_c)]^{\frac{1}{D_s+1}} \\ &= \frac{1}{C} (\phi_c^{2-n} - \phi_i^{2-n}) + \frac{4\pi G}{\nu} (\phi_e^2 - \phi_c^2), \end{aligned} \quad (4.44)$$

for $m > 2, \nu \neq m$ and

$$\begin{aligned} N &= \frac{1}{n(\nu^2 - 4)} \log [n(\nu^2 - 4)H_i^2(t_c - t_i)] \\ &\quad - \frac{B_s}{C_s} + \frac{1}{C_s} [B_s^{D_s+1} + A_s C_s (D_s + 1)(t_e - t_c)]^{\frac{1}{D_s+1}} \\ &= \frac{1}{m(2 - \nu)} \log \left(\frac{\phi_i}{\phi_c} \right) + \frac{4\pi G}{\nu} (\phi_e^2 - \phi_c^2), \end{aligned} \quad (4.45)$$

for $n = 2, \nu \neq 2$. Note that the first terms of the above equations arise from the Lorentz violating stage. As an example, let us take the values: $N = 70$, $m = 10^{-2}$, $n = 2$ and $\nu = 1$. If $\phi_e \sim 0.3M_{pl}$ is the value of scalar field at the end of inflation, then, $\phi_c \sim 2M_{pl}$. The contribution from the inflation end is still relevant. Therefore, we get $\phi_i \sim 2.5M_{pl}$.

4.2 Power law potential: $V(\phi) = \frac{1}{2}M^2\phi^2$

4.2.1 Lorentz violating stage

The most realistic inflationary universe scenarios are chaotic models. For the model $V(\phi) = \frac{1}{2}M^2\phi^2$, assuming the slow roll conditions, we find the slow roll equations during the Lorentz

violating regime as follows

$$H^2 = \frac{M^2}{6n} \phi^{-(n-2)}, \tag{4.46}$$

$$\phi' = -m(n+2)\phi^{(n-1)}. \tag{4.47}$$

Then we find the solution (4.47) as

$$\phi(\alpha) = [\phi_i^{2-n} + m(n^2 - 4)(\alpha - \alpha_i)]^{\frac{1}{2-n}}, \tag{4.48}$$

for $m \neq 2$ and

$$\phi(\alpha) = \phi_i e^{-4m(\alpha - \alpha_i)}, \tag{4.49}$$

for $n = 2$. The inflationary scenario of this model was already obtained in ref. [24] where the Hubble parameter becomes constant during the Lorentz violating regime and $1/226 < m < 1/16$ is the range of parameter m . We concern here the solution for $n \neq 2$. The solution for the Hubble parameter is given by

$$H^2(\alpha) = \frac{M^2}{6m} [\phi_i^{2-n} + m(n^2 - 4)(\alpha - \alpha_i)], \tag{4.50}$$

and

$$\frac{a(t)}{a_i} = \exp \left[\frac{1}{m(n^2 - 4)} \left(\frac{1}{\phi^{n-2}(t)} - \frac{1}{\phi_i^{n-2}} \right) \right], \tag{4.51}$$

which is the solution for the scale factor. As in the previous subsection, we also obtain $n > 2$ which the effect of Lorentz violation occurs in this regime. The time evolution of the above equations can be obtained by integrating eq. (4.50), we get

$$\alpha(t) = \alpha_i - \frac{b}{c} + \frac{1}{c} \left[b^{1/2} + \frac{1}{2} dc(t - t_i) \right]^2, \tag{4.52}$$

where

$$b = \phi_i^{2-n}, \quad c = m(n^2 - 4), \quad d = \sqrt{\frac{M^2}{6m}}, \quad n > 2. \tag{4.53}$$

Then the evolution equations are given by

$$\frac{a(t)}{a_i} = \exp \left\{ -\frac{b}{c} + \frac{1}{c} \left[b^{1/2} + \frac{1}{2} dc(t - t_i) \right]^2 \right\}, \tag{4.54}$$

$$\phi(t) = \left[b^{1/2} + \frac{1}{2} dc(t - t_i) \right]^{-\frac{2}{n-2}}, \tag{4.55}$$

$$H(t) = d \left[b^{1/2} + \frac{1}{2} dc(t - t_i) \right], \tag{4.56}$$

$$\beta(t) = n \left[b^{1/2} + \frac{1}{2} dc(t - t_i) \right]^{-\frac{2n}{n-2}}, \tag{4.57}$$

$$\rho(t) = 3nd^2 \left[b^{1/2} + \frac{1}{2} dc(t - t_i) \right]^{-\frac{4}{n-2}}. \tag{4.58}$$

Since b , c and d are positive constants, one can see that the Hubble parameter H and the scale factor a increase during the Lorentz violating stage for $n > 2$. In the case $n = 2$, the Hubble parameter is constant. In the following subsection, we will see that the Hubble parameter decreases in the standard slow roll stage. Therefore, we can generate the spectrum with the initial (steep) blue and the later (slightly) red spectra. This may explain the deficiency of the CMB power spectrum at large scales observed by WMAP [11].

4.2.2 Standard slow roll stage

Now, let us consider the chaotic inflationary scenario in the standard slow roll stage. A set of the dynamical equations of the scalar field are given by eqs. (4.32)–(4.34). Assuming the standard slow roll conditions, we find the slow roll equations

$$H^2 \simeq \frac{4\pi G}{3} M^2 \phi^2, \tag{4.59}$$

$$\phi' \simeq -\frac{1}{4\pi G} \phi^{-1}. \tag{4.60}$$

The evolution of the inflaton can be solved as

$$\phi^2(\alpha) = \phi_c^2 - \frac{1}{2\pi G}(\alpha - \alpha_c). \tag{4.61}$$

The Hubble parameter and the scale factor $a(t) = e^\alpha$ can be also obtained as

$$H^2 = \frac{4\pi G M^2}{3} \left(\phi_c^2 - \frac{1}{2\pi G}(\alpha - \alpha_c) \right), \tag{4.62}$$

$$a(t) = a_c \exp [2\pi G(\phi_c^2 - \phi^2(t))]. \tag{4.63}$$

From eq. (4.62), we obtain

$$\alpha(t) - \alpha_c = \frac{b_s}{c_s} - \frac{1}{c_s} \left[b_s^{1/2} - \frac{1}{2} d_s c_s (t - t_c) \right]^2, \tag{4.64}$$

and the dynamical evolutions are given by

$$\frac{a(t)}{a_c} = \exp \left[\frac{b_s}{c_s} - \frac{1}{c_s} \left(b_s^{1/2} - \frac{1}{2} d_s c_s (t - t_c) \right)^2 \right], \tag{4.65}$$

$$\phi(t) = b_s^{1/2} - \frac{1}{2} d_s c_s (t - t_c), \tag{4.66}$$

$$H(t) = d_s \left[b_s^{1/2} - \frac{1}{2} d_s c_s (t - t_c) \right], \tag{4.67}$$

$$\rho(t) = \frac{3c_s d_s}{4} \left[b_s^{1/2} - \frac{1}{2} d_s c_s (t - t_c) \right]^2, \tag{4.68}$$

with

$$b_s = \phi_c^2, \quad c_s = \frac{1}{2\pi G}, \quad d_s = \frac{4\pi G M^2}{3}. \tag{4.69}$$

Note that the Hubble parameter decreases in the standard slow roll stage.

In the case of chaotic potential, the total e-folding number reads

$$\begin{aligned}
 N &= -\frac{b}{c} + \frac{1}{c} \left[b^{1/2} + \frac{1}{2}dc(t_c - t_i) \right]^2 + \frac{b_s}{c_s} - \frac{1}{c_s} \left[b_s^{1/2} - \frac{1}{2}d_s c_s(t_e - t_c) \right]^2 \\
 &= \frac{1}{m(n^2 - 4)} (\phi_c^{2-n} - \phi_i^{2-n}) + 2\pi G (\phi_c^2 - \phi_e^2), \tag{4.70}
 \end{aligned}$$

where ϕ_e is the value of scalar field at the end of inflation. The first term arises from the Lorentz violating stage.

5. Phase-space analysis

In this section, we investigate the global structure of the dynamical system via phase plane analysis and compute the cosmological evolution by numerical analysis. Introducing the following variables:

$$x \equiv \frac{\phi'}{\sqrt{6\beta}}, \quad y \equiv \sqrt{\frac{V}{3H^2\beta}}, \tag{5.1}$$

$$\lambda_1 \equiv -\frac{\bar{\beta}_{,\phi}}{\sqrt{\bar{\beta}}}, \quad \lambda_2 \equiv -\sqrt{\bar{\beta}} \frac{V_{,\phi}}{V}, \tag{5.2}$$

$$\Gamma_1 \equiv \frac{\bar{\beta}\bar{\beta}_{,\phi\phi}}{\bar{\beta}_{,\phi}^2}, \quad \Gamma_2 \equiv \frac{VV_{,\phi\phi}}{V_{,\phi}^2}, \tag{5.3}$$

the eqs. (4.2) and (4.3) can be written as a plane-autonomous system

$$x' = -3x(1 - x^2) + \sqrt{\frac{3}{2}}(\lambda_1 + \lambda_2)y^2, \tag{5.4}$$

$$y' = \left[3x - \sqrt{\frac{3}{2}}(\lambda_1 + \lambda_2) \right] xy, \tag{5.5}$$

$$\lambda_1' = -\sqrt{6}\lambda_1^2 \left(\Gamma_1 - \frac{1}{2} \right) x, \tag{5.6}$$

$$\lambda_2' = -\sqrt{6}\lambda_2^2 \left[\Gamma_2 - \left(1 - \frac{\lambda_1}{2\lambda_2} \right) \right] x, \tag{5.7}$$

where the prime denotes a derivative with respect to the logarithm of the scale factor, $\alpha = \ln a$. The functions $\lambda_1(\phi)$ and $\lambda_2(\phi)$ determine a type of the coupling vector and the potential, respectively. The Friedmann constraint, eq. (4.1), takes the simple form

$$x^2 + y^2 = 1. \tag{5.8}$$

The equation of state for the scalar field could be expressed in terms of the new variables as

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{x^2 - y^2}{x^2 + y^2}. \tag{5.9}$$

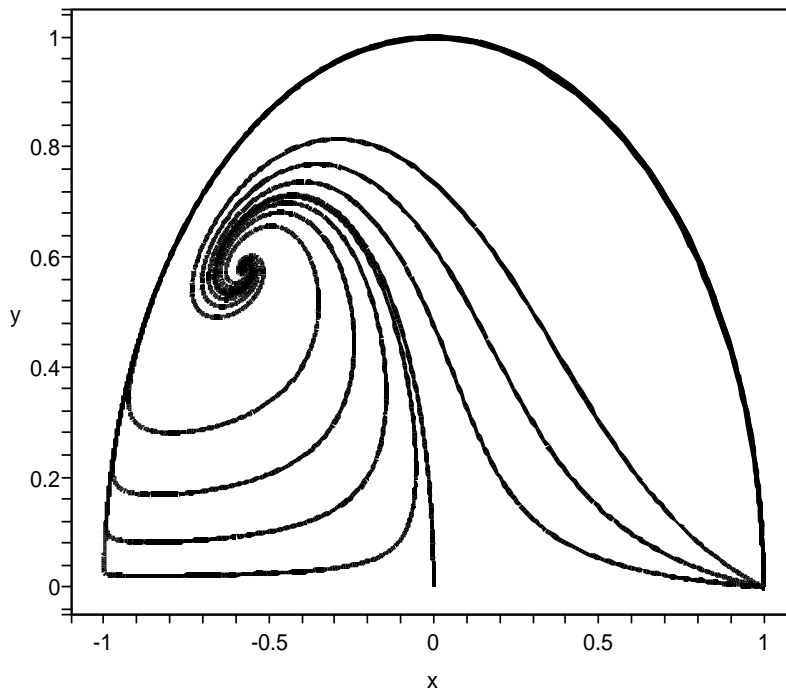


Figure 1: The phase plane of Lorenz violating kinetic dominated solution for $m > 3/8$.

Notice that x^2 measures the contribution to the expansion due to the scalar field kinetic energy and the coupling function, while y^2 measures the contribution to the expansion due to the potential energy and the the coupling function.

Equations (5.4)–(5.7) are written as an autonomous phase system of the form $\mathbf{x}' = \mathbf{f}(\mathbf{x})$ where $\mathbf{x} = (x, y, \lambda_1, \lambda_2)$. The use of this form for the dynamical equations allows the fixed points of the system to be readily identified, and the so-called critical points \mathbf{x}_0 are solutions of the system of equations $\mathbf{f}(\mathbf{x}_0) = 0$. To determine their stability we need to perform linear perturbations around the critical points in the form $\mathbf{x} = \mathbf{x}_0 + \mathbf{u}$, which results in the following equations of motion $\mathbf{u}' = M\mathbf{u}$, where

$$M_{ij} = \left. \frac{\partial f_i}{\partial x_j} \right|_{\mathbf{x}_0} . \tag{5.10}$$

In the case of the dynamical equations (5.4)–(5.7), \mathbf{u} is a 4-column vector consisting of the perturbations of x , y , λ_1 and λ_2 . Thus, M_{ij} is a 4×4 matrix. The stability of the critical points is determined by the eigenvalues μ_i of the matrix M at the critical points. A non-trivial critical point is called stable (unstable) whenever the eigenvalues of M are such that $Re(\mu_i) < 0$ ($Re(\mu_i) > 0$). If neither of the aforementioned cases are accomplished, the critical point is called a saddle point.

In the following, we will study the simplest model,

$$\bar{\beta}(\phi) = m\phi^2, \quad V(\phi) = \frac{1}{2}M^2\phi^2, \tag{5.11}$$

where m and M are parameters. Substituting eqs. (5.11) into eqs. (5.2) and (5.3), respectively, we obtain

$$\lambda_1 = \lambda_2 = -2\sqrt{m}, \quad \Gamma_1 = \Gamma_2 = \frac{1}{2}, \quad (5.12)$$

and eqs. (5.6) and (5.7) are trivially satisfied. In the former, eqs. (5.4) and (5.5) can be fused into the single equation,

$$\begin{aligned} x' &= - \left[3x - \sqrt{\frac{3}{2}}(\lambda_1 + \lambda_2) \right] (1 - x^2) \\ &= - (3x + 2\sqrt{6m}) (1 - x^2), \end{aligned} \quad (5.13)$$

which is one dimensional phase-space corresponding to the unit circle. Critical points correspond to fixed points where $\mathbf{x}' = 0$, and there are Lorentz violation self-similar solutions with

$$\frac{H'}{H} = -3x^2 + \sqrt{6}\lambda_1 x. \quad (5.14)$$

Note that the second term arises from Lorentz violation. Applying the above procedure, setting $\mathbf{x}' = 0$, the critical points (x_0, y_0) of the system are $(1, 0)$, $(-1, 0)$, and $(-\sqrt{8m/3}, \sqrt{1 - 8m/3})$. For any form of the potential in the Lorentz violation stage, the critical points $(1, 0)$ or $(-1, 0)$ correspond to two Lorentz violation kinetic-dominated solutions. Then, the critical point $(-\sqrt{8m/3}, \sqrt{1 - 8m/3})$ corresponds to a Lorentz violation potential-kinetic solution. Integration of eq. (5.14) with respect to α will show that all critical points, \mathbf{x}_0 , correspond to the Hubble parameter

$$H \propto \exp\left(-\frac{\alpha}{p}\right). \quad (5.15)$$

This relates to an expanding universe with a scale factor $a(t)$ given by $a(t) \sim t^p$, where

$$p \equiv \frac{1}{3x_0^2 - \sqrt{6}\lambda_1 x_0} = \frac{1}{3x_0^2 + 2\sqrt{6m}x_0}. \quad (5.16)$$

The linear perturbation about the points $x_{0+} = +1$ and $x_{0-} = -1$ give the eigenvalues $\mu_+ = 6 + 4\sqrt{6m}$ and $\mu_- = 6 - 4\sqrt{6m}$, respectively. Thus for positive m , $x_{0+} = +1$ is always unstable and $x_{0-} = -1$ is stable for $m > 3/8$ but unstable for $m < 3/8$. Moreover, in the linear perturbation about the Lorentz violation potential-kinetic solution, we obtain the eigenvalue $\mu = 8m - 3$. The solution is stable for $m < 3/8$. In figures 1 and 2, we show the phase plane plot for $m > 3/8$ and $m < 3/8$. We note that the trajectories are confined inside the circle given by $x^2 + y^2 = 1$.

Another remarkable feature of the above model is that the equation of state is given by

$$\omega_\phi = -1 + \frac{16}{3}m, \quad (5.17)$$

completely determined by the parameter m of the coupling vector. Thus, we always have $\omega_\phi > -1$ for ordinary scalar field.

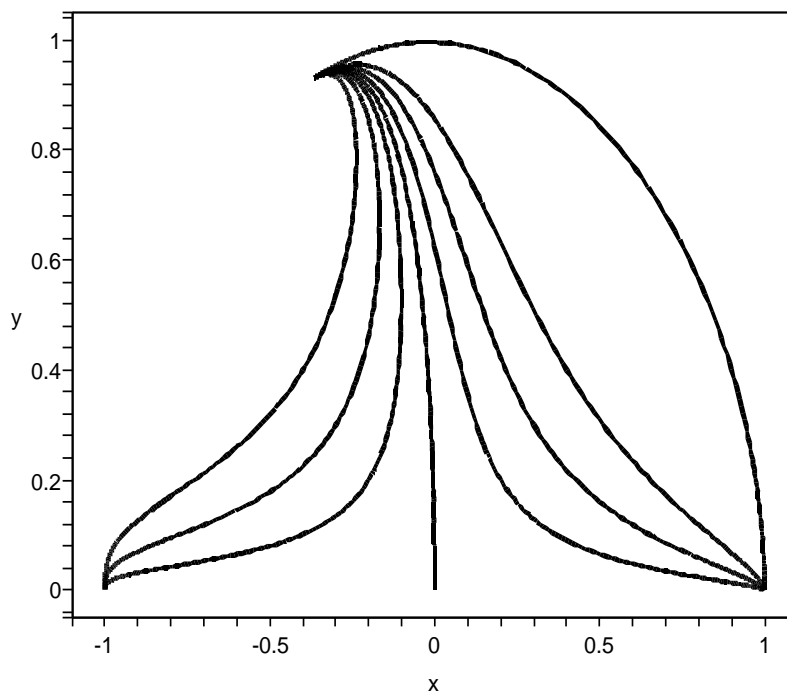


Figure 2: The phase plane of Lorentz violating kinetic-potential solution for $m < 3/8$.

6. Conclusions

In this paper, we have studied the dynamics of a scalar field in the Lorentz violating scalar-vector-tensor theory of gravity, taking into account the effect of the power-law effective coupling vector. Since the effective coupling vector be dynamics variable, the equation of state is dependent on the coupling parameter. For the model with the power law Hubble parameter and coupling vector, we find an exact solution of the equation of state. A constant equation of state corresponds to $n = 2$ while for $n > 2$ leads to dynamics equation of states. In this case, the scalar fields are completely associated with the Lorentz violation.

Also, the different form in the coupling vector and the potential models lead to the different qualitative evolution in two regimes of inflation. The results show that, for the inverse power-law potential, the Hubble parameter decreases during the Lorentz violation stage and increases in the standard slow roll stage. For the power-law potential, the Hubble parameter increases during the Lorentz violation stage but it decreases in the standard slow roll stage.

From the qualitative study of the dynamical system, we have demonstrated the attractor behavior of inflation driven by a scalar field in the context of scalar-vector-tensor theory of gravity. We have found that there exists the Lorentz violating kinetic dominated solution and the Lorentz violating potential-kinetic dominated solution, depending on the region of the coupling parameter in the simplest Lorentz violating chaotic inflation

model. The quadratic coupling vector and the chaotic potential correspond to the constants $\lambda_1 = \lambda_2 = -2\sqrt{m}$ and $\Gamma_1 = \Gamma_2 = 1/2$. There are two important results of this study, which are different from the scalar-tensor theory of gravity: the condition for the accelerating universe, eq. (5.14) and the slope, p , eq. (5.16). The first one yields $\lambda_1 \mathbf{x}_0 > \sqrt{3/8}(\omega_\phi + 1/3)$. The analysis of the critical points show that we may obtain an accelerated expansion provided that the solutions are approaching the Lorentz violation kinetic dominated solution with $m > 1/6$ and approaching the Lorentz violation potential-kinetic dominated solution with $m < 3/8$. When the accelerating condition is satisfied, the slope p characterizes the properties of the inflating universe: power-law inflation ($p > 0$), de Sitter inflation ($p = 0$) and superinflation ($p \equiv -|p| < 0$). In other cases, if λ_1 and λ_2 are constants, one finds that the coupling vector is still quadratic in scalar field, $\bar{\beta} \sim \phi^2$, while the potential as a function of scalar field ϕ is given by a power-law potential, $V(\phi) \sim \phi^{2\gamma}$ and $\Gamma_1 = 1/2$, $\Gamma_2 = 1 - 1/2\gamma$, where $\gamma = \lambda_2/\lambda_1$. Moreover, in order to obtain dynamical evolution of the system, we need to solve eqs. (5.6) and (5.7) together with eqs. (5.4) and (5.5). For a realistic model, the effect of an additional component (matter field) would be interesting [38].

Finally, we would like to emphasize that there exists an attractor solution in the Lorentz violating scalar-vector-tensor theory of gravity.

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